Physics Notes

BY

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Class:10+1
Unit: II
Topic: Kinematics

SYLLABUS: UNIT-II-C,D

Frame of reference. Motion in a straight line: Position-time graph, speed and velocity, Uniform and non-uniform motion, average speed and instantaneous velocity.
Uniformly accelerated motion, velocity-time, position-time graphs, relations for uniformly accelerated motion (graphical treatment).
Elementary concept of differentiation and integration for describing motion.
Scalar and vector quantities: Position and displacement vectors, general vectors and notation, equality of vectors, multiplication of vectors by a real number; addition and subtraction of vectors. Relative velocity.
Unit vector; Resolution of a vector in a plane – rectangular components, Motion in a plane, Cases of uniform velocity and uniform acceleration-projectile motion. Uniform circular motion.

Unit 2-C Vectors

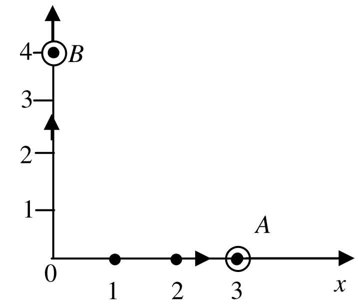
Q.1. Explain position and displacement vector? With Example.

Ans. **Position Vector**:-

A vector used to represent position of a point/particle is called position vector.

Example 1:- $\overrightarrow{OA} = 3\,\hat{\imath}$ Is position vector of point *A*.

Example 2:- $\overrightarrow{OB} = 4\,\hat{\jmath}$ Is position vector of point *B*.



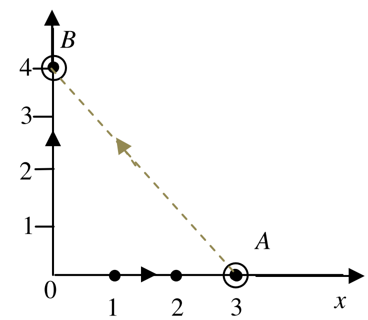
Displacement Vector:-

Displacement vector is a vector marked from initial position to final position of a particle or object.

Example:- A → initial position

B → final position

$\overrightarrow{AB}$ → displacement vector



Q2. Explain the following with examples:

i) Unit Vector

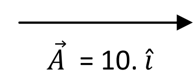
ii) Equal Vector

iii) Multiplication of a vector by real number

Ans.i) **<u>Unit Vector</u>**:-

A Vector having magnitude unity is called a unit vector.
It is denoted by $\hat{A}$

$$\hat{A} = \frac{\vec{A}}{|\vec{A}|}$$

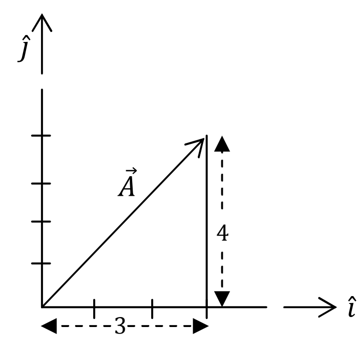


Example1: $\vec{A} = 10.\,\hat{\imath}$

$|\vec{A}| = 10$

$\hat{A} = \frac{10.\hat{\imath}}{10}$

$= 1.\,\hat{\imath}$



$\hat{\imath}$ → Unit vector along x-axis
$\hat{\jmath}$ → Unit vector along y-axis

Example2: $\vec{A} = 3.\,\hat{\imath} + 4.\,\hat{\jmath}$

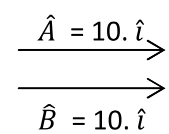
$|\vec{A}| = \sqrt{(3)^2 + (4)^2} = 5$

$\hat{A} = \frac{\hat{A}}{|\vec{A}|}$

$$\boldsymbol{\hat{A} = \frac{3.\hat{\imath} + 4.\hat{\jmath}}{5}}$$

ii) **<u>Equal Vectors</u>**:-

Two vectors $\vec{A}$ and $\vec{B}$ are said to be equal vectors, if they have equal magnitude and same direction.

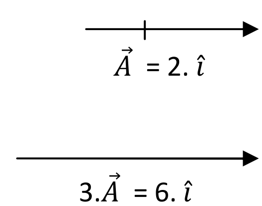


Example:- $\vec{A} = 10.\,\hat{\imath}$, $\vec{B} = 10.\,\hat{\imath}$

These two vectors $\vec{A}$ and $\vec{B}$ are equal.

iii) **<u>Multiplication of a vector by real number</u>**:-

If a vector is multiplied by a number *n*, magnitude of vector becomes *n* times but direction remains same.



Example:- $\vec{A} = 2.\,\hat{\imath}$

And $3(\vec{A}) = 3(2.\,\hat{\imath})$

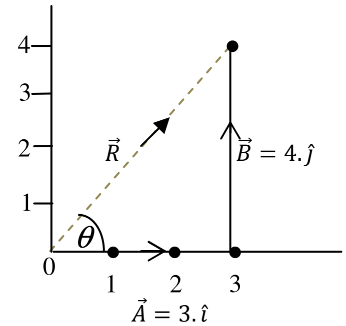
$= 6.\,\hat{\imath}$

Q.3. **a) State triangle law of vector addition?**

b) Find $|\vec{R}|$ and direction of $\vec{R}$ resultant?

Ans.a) **<u>Triangle Law</u>**:-

If two vectors acting on a particle are taken as two sides of a triangle in one order, then third side of triangle taken in opposite order gives resultant vector.



<u>Example</u>:- $\vec{A} = 3.\hat{\imath}$ $\quad \vec{B} = 4.\hat{j}$

$\vec{R} = \vec{A} + \vec{B}$

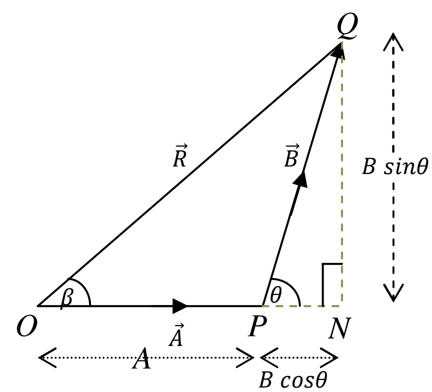
$= 3.\hat{\imath} + 4.\hat{j}$

$(\vec{R})$ = 5, Direction as shown in Fig. Making angle θ with x-axis

b) **<u>Proof</u>**:-

$|\vec{R}| = ?$

$|\vec{R}| = \sqrt{A^2 + B^2 + 2.A.B.cos\theta}$



$(OQ)^2 = (ON)^2 + (QN)^2$

$= (OP + PN)^2 + (QN)^2$

$= (A + B.cos\theta)^2 + (B.sin\theta)^2$

$= A^2 + B^2 cos^2\theta + 2AB\ cos\theta + B^2 sin^2\theta$

$= A^2 + B^2 (cos^2\theta + sin^2\theta) + 2AB\ cos\theta$

$= A^2 + B^2 + 2AB\ cos\theta$

$OQ = \sqrt{A^2 + B^2 + 2.A.B.cos\theta}$

$$|\vec{R}| = \sqrt{A^2 + B^2 + 2.A.B.cos\theta}$$

In Δ QON, tan β $= \frac{QN}{ON}$

$= \frac{B\ sin\theta}{OP + PN}$

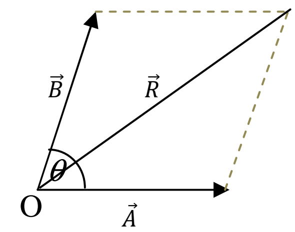
$$\textbf{tan } \boldsymbol{\beta} = \frac{\boldsymbol{B\ sin\theta}}{\boldsymbol{A + B\ cos\theta}}$$

- β is the angle which resultant makes with vector A.

Q.4. **a) State Parallelogram law of vector addition?**
b) Find $|\vec{R}|$ and direction of resultant $\vec{R}$?

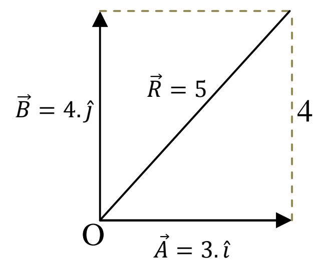
Ans.a) **Parallelogram Law**:-

It states that if two vectors acting on a particle at the same time be represented in magnitude and direction by the two adjacent sides of parallelogram drawn from a point, their resultant vector is represented in magnitude and direction by the diagonal of parallelogram drawn from the same point.



Example:- $\vec{A} = 3.\hat{\imath}$, $\vec{B} = 4.\hat{\jmath}$

$|\vec{R}| = 5$, direction as shown in Fig.



b) **Proof**:-

Draw $QN \perp ON$ extended line OP to N

$\therefore QN \perp ON$

In Δ OQN,

$(OQ)^2 = (QN)^2 + (ON)^2$

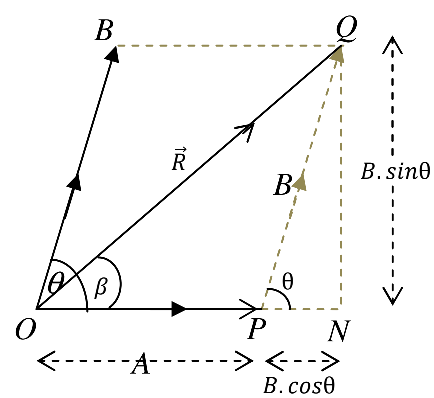
$= (B.sin\theta)^2 + (OP + PN)^2$

$= (B.sin\theta)^2 + (A + B.cos\theta)^2$

$= B^2\ sin^2\theta + A^2 + B^2\ cos^2\theta + 2AB\ cos\theta$

$OQ = \sqrt{A^2 + B^2 + 2.A.B.cos\theta}$

$$|\vec{R}| = \sqrt{A^2 + B^2 + 2.A.B.cos\theta}$$



In Δ QON, tan $\beta = \frac{QN}{ON}$

$= \frac{B\ sin\theta}{OP + PN}$

$$\tan \beta = \frac{B\ sin\theta}{A+B\ cos\theta}$$

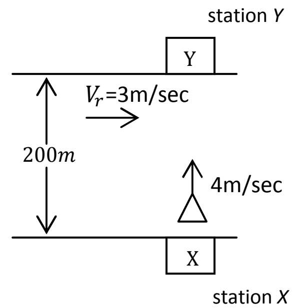
- β is the angle which resultant makes with vector A.

Q5. Discuss crossing the river

a) Shortest time

b) Shortest path.

Ans. **Concept Example1**:-



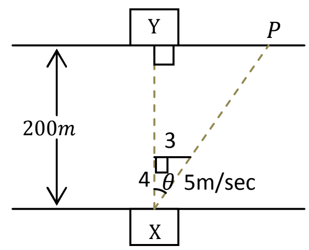
Width of river	= 200m
Boat's speed in still water	= 4 m/sec
River's speed	= 3m/sec

i) Find time taken by the boat to cross the river?

ii) Find time taken by the man to reach station Y, along the bank, if walking speed of the person is 6m/sec?

Sol:-

i) Let V_b= 4m/sec, V_r = 3 m/sec and $\theta = 90^\circ$



∴ Resultant, $|\vec{R}| = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

$V_{resultant} = \sqrt{(4)^2 + (3)^2 + 2.4.3\cos 90}$

$= \sqrt{16 + 9}$

$V_{resultant} = \sqrt{25}$ = 5m/sec

∴ Tan θ = $\frac{3}{4} = \frac{YP}{XY}$

⇒ $\frac{3}{4} = \frac{YP}{200}$

⇒ YP = 150 m

∴ In rt. Angled ΔXYP

XP $= \sqrt{(150)^2 + (200)^2}$

XP = 250 m

∴ Time taken to cross river,

t $= \frac{XP}{V_{resultant}}$

$= \frac{250}{5}$

t = 50 sec

<u>Alternate</u>

Time, t = $\frac{displacement\ along\ Y-axis}{velocity\ along\ Y-axis}$

$= \frac{200\ m}{4\ m/sec}$

= 50 sec

ii) Distance travelled by walking = 150m (YP = 150m)

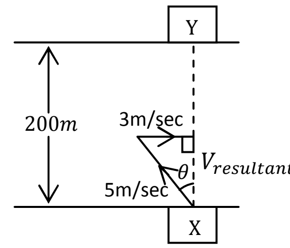
Velocity of person = 6 m/sec

Time taken, t $= \frac{150}{6}$

= 25 sec

Concept Example2:-

Width of river = 200m
Boat's speed = 5m/sec
River's speed = 3m/sec



i) Find the angle, θ at which boat should move so that it reached exactly the opposite station Y.

ii) Find time taken by the boat to cross the river.

Sol:-

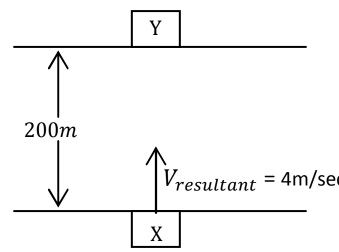
i) Sin θ = $\frac{3}{5}$

∴ θ = $\left(\frac{3}{5}\right) sin^{-1}$

ii) Resultant velocity, $|\vec{V}_{res}|$ = $\sqrt{25-9}$

= $\sqrt{16}$

= 4 m/sec



Distance = 200m

Velocity = 4m/sec

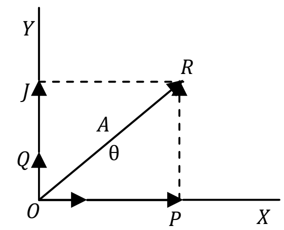
∴ Time taken = $\frac{200}{4}$

= 50sec

Q6. Explain rectangular components of a vector in 2-Dimension and 3-Dimension.

Ans.a) **RECTANGULAR COMPONENTS OF A VECTOR IN A PLANE**

When a vector is splitted into two component vector at right angles to each other, the component vector are called rectangular components of a vector.



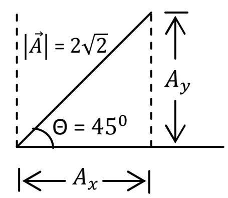
A_x = A cos θ

A_y = A sin θ

A = $\sqrt{A_x^2 + A_y^2}$

b) Example:-

$|\vec{A}|$ = $2\sqrt{2}$ at θ = 45° with x-axis Find A_x and A_y



Sol:- $\frac{A_x}{A}$ = cos θ

A_x = A cos θ

= $2\sqrt{2}$. cos 45

= 20

$\frac{A_y}{A}$ = sin θ

A_y = A sin θ

= $2\sqrt{2}$. sin 45

= $2\sqrt{2} \cdot \frac{1}{\sqrt{2}}$

= 2

c) **RECTANGULAR COMPONENTS OF A VECTOR IN THREE DIMENSIONS.**

A = $A_x i + A_y j + A_z k$

A = $\sqrt{A_x^2 + A_y^2 + A_z^2}$

If α, β and γ are the angles which the $\vec{A}$ makes with X, Y, and Z axes respectively, then

Cos α = A_x/A or A_x = A cos α

Cos β = A_y/A or A_y = A cos β

Cos γ = A_z/A or A_z = A cos γ

Here cos α, cos β and cos γ are called the direction cosines of the vector A

$\cos^2 α + \cos^2 β + \cos^2 γ = 1$

Unit 2-D

1 **Q1.** **What is a projectile? A particle is projected horizontally with a speed *u* from a building of height *h*.**

- **a) Prove path followed by projectile is parabola.**
- **b) Find time of flight.**
- **c) Find horizontal range.**
- **d) Find velocity of particle at any instant of time.**

Ans. **PROJECTILE**

Projectile is a body thrown with some initial velocity with the horizontal direction, and then allowed to move under the action of gravity alone, it's path is called ***trajectory***.

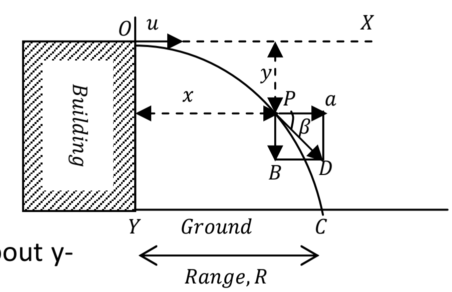
Projectile moves under the combined effect of:

1. *A uniform velocity in the horizontal direction, which would not change.*
2. *A uniformly changing velocity (i.e. increasing or decreasing) in the vertical direction due to gravity.*

X-axis analysis is independent of Y-axis analysis.

PROJECTILE GIVEN HORIZONTAL PROJECTION

a)
1. *Horizontal velocity u,* which remains constant throughout the motion (neglecting the air friction).
2. *Vertical velocity* which increases due to gravity. Initial value of this velocity at O is zero.



Path of projectile

$$x = 0 + ut + \frac{1}{2}(0)t^2 = ut \quad \text{------ (1)}$$

$$y = 0 + (0)\,t + \frac{1}{2}gt^2 = \frac{1}{2}gt^2 \quad \text{------ (2)}$$

$$\boxed{y = k\,x^2} \quad \text{------ (3)} \quad \text{(Eliminating } t\text{)}$$

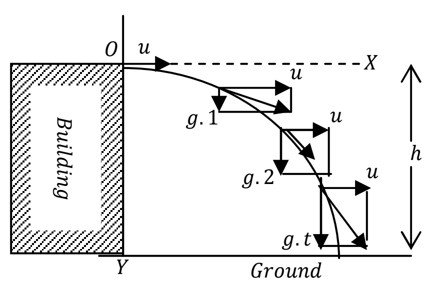
This is an equation of a parabola, which is symmetrical about y-axis.

b) **Time of Flight**

As $y = y_0 + u_y t + \frac{1}{2}a_y t^2$

$$h = 0 + 0 \times T + \frac{1}{2}gT^2$$

Or $\boxed{\mathbf{T} = \sqrt{2h/g}}$ ------ (4)



c) **Horizontal range.**

As $x = x_0 + u_x t + \frac{1}{2}a_x t^2$

$$R = 0 + u\sqrt{2h/g} + \frac{1}{2} \times 0 \times T^2$$

$\boxed{\mathbf{R} = \mathbf{u}\sqrt{2h/g}}$ ------ (5)

d) **Velocity of the object (i.e. projectile) at any instant.**

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{u^2 + g^2 t^2}$$

$\boxed{v = \sqrt{u^2 + g^2.t^2}}$ ------ (6)

Q2. A particle is projected with initial speed u making angle θ with the horizontal.

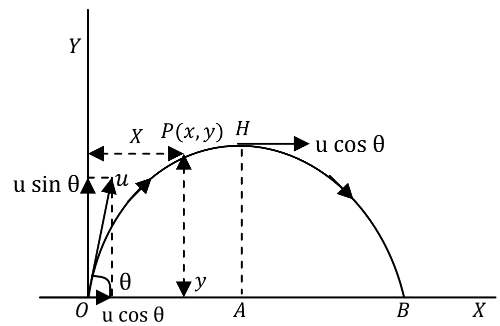
a) Prove path followed by projectile is parabola.
b) Find time of flight.
c) Find horizontal range.
d) Find maximum height reached.

Ans

a) **Motion along horizontal direction, OX**

$x = 0 + (u\cos\theta)\,t + \frac{1}{2}(0)\,t^2 = u\cos\theta\, t$

or $\boxed{t = \frac{x}{u\cos\theta}}$ ------ (1)



Motion along vertical direction, OY

$y = 0 + u\sin\theta\, t + \frac{1}{2}(-g)\,t^2$

Or $\boxed{y = (u\sin\theta)\,t - \frac{1}{2}gt^2}$ ------ (2)

Eliminating *t* from (1) and (2)

$y = u\sin\theta\left(\frac{x}{u\cos\theta}\right)\ \frac{1}{2}g\left(\frac{x}{u\cos\theta}\right)^2$

Or $\boxed{\mathbf{Y = x\tan\theta - \frac{1}{2}\left(\frac{g}{u^2\cos^2\theta}\right)x^2}}$

This is an equation of a parabola.

b) **Time of flight, *T***

It is the total time for which the object is in flight (i.e. remains in air), while going from *O* to *B*. It is denoted by *T*.

Time of ascent = time of descent = t (say)
As, total time of flight = time of ascent + time of descent

$T = t + t = 2t$

Or $t = T/2$

Motion along Y-axis $V_y = u_y + a_y t$

$0 = u\sin\theta + (-g)\,T/2$

$\boxed{T = \frac{2u\sin\theta}{g}}$

c) **Horizontal Range, *R***

It is the horizontal distance covered by the object between its point of projection and the point of hitting the ground. It is denoted by *R*.

$R = u\cos\theta \times T = u\cos\theta \times \frac{2u\sin\theta}{g}$

$= \frac{u^2}{g}\,2\sin\theta\cos\theta$

$\boxed{\mathbf{R = \frac{u^2\sin 2\theta}{g}}}$

d) **Maximum height, *H***

It is the maximum vertical height attained by the object above the point of projection during its flight.

$H = 0 + (u\sin\theta) \times \frac{u\sin\theta}{g} + \frac{1}{2}(-g)\left(\frac{u\sin\theta}{g}\right)^2$

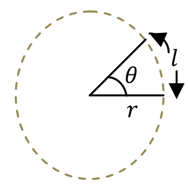
Or $h = \frac{u^2}{g}\sin^2\theta - \frac{1}{2}\,\frac{u^2\sin^2\theta}{g} = \frac{u^2\sin^2\theta}{2g}$

$\boxed{\mathbf{H = \frac{u^2\sin^2\theta}{2g}}}$

Q3. a) Define Angular Displacement? Units? Dimensions?
b) Define Angular Velocity? Units? Dimensions? With examples.

Ans.

a) **Angular Displacement**:- Angle traced by radius vector *r*.



Angular Displacement, $\Theta = \frac{l}{r}$

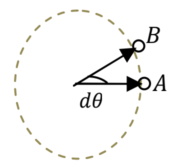
$l \rightarrow$ length of arc
r $\rightarrow$ radius of circle

Units:- S.I. Unit → radians (rad)

Dimensions:- $[\Theta] = \frac{[l]}{[r]} = \frac{[L]}{[L]}$

$[\Theta] = [M^0 L^0 T^0]$ Dimensionless

b) **Angular Velocity**:-

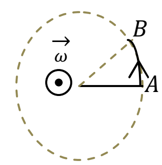


"Time rate of change of Angular displacement".

Angular Speed, $\omega = \frac{d\theta}{dt}$

Angular velocity, $\vec{\omega} = \frac{\overrightarrow{d\theta}}{dt}$

$\vec{\omega}$ away from reader when particle moves in anticlockwise direction

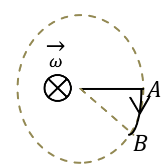


Direction:- $\vec{\omega}$ Axial Vector

Towards the reader for ACW.

Away from the reader for CW.

$\vec{\omega}$ towards the reader when particle moves in clockwise direction



Units:- S.I. Units $\rightarrow \frac{rad}{sec}$

Dimensions:- $[\omega] = [M^0\ L^0\ T^{-1}]$

Example. Find angular speed of hour hand of a clock.

Sol. Angular covered by hour hand,

$\theta = 2\pi$ rad (360°)

Time taken by hour hand to complete

One circle = 12 hrs

Angular speed, $\omega = \frac{\theta}{t}$

$= \frac{2\pi \text{ rad}}{12\ hrs}$

$= \frac{\pi}{6}$ rad/hr

Problem. **Find angular speed of minute hand of a clock?**

Ans. (Ask your friend for the same)

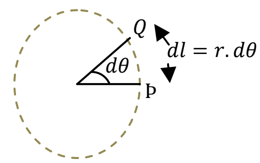
Q4. **Relationship between linear velocity v and angular velocity ω? (Scalar Relation).**

Ans. A particle moving from P to Q covering angle $d\theta$ (in time dt)

$v = \frac{dl}{dt}$ and $\omega = \frac{d\theta}{dt}$

$v = \frac{dl}{dt}$

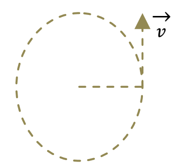
$v = r\left(\frac{d\theta}{dt}\right)$ $\left\{d\theta = \frac{dl}{r}\right\}$



$$\boxed{v = r.\omega}$$

Direction of $\vec{V}$ is tangent to the circle

Q5. **a) Define "Angular Acceleration"?**
b) Relation between "Linear Acceleration" and "Angular Acceleration"?

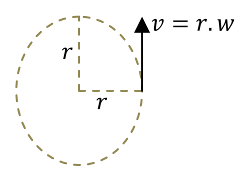


Ans.a) **<u>Angular Acc</u>**:-

Angular acc is defined as rate of change of angular velocity w.r.t time.

$\alpha = \frac{d\omega}{dt}$

<u>Relation between a and ∝</u>



acc, a $= \frac{dv}{dt}$

a $= \frac{d}{dt}$ (r. ω) [As v = r. ω]

a $= \left(\frac{d\omega}{dt}\right)$ r

a $= \alpha$. r

$$\boxed{\mathbf{a = r\,.\,\alpha}}$$

<u>Units</u>:- rad/sec^2

<u>Dimensions</u>:- $[\alpha] = [M^0\ L^0\ T^{-2}]$

Q6. Define Time Period *T*, Frequency *f*. Write relation between *T* and *f*, relation between ω and *f*.

Ans.

$$T = 1/f$$

$$f = 1/T$$

$$f \cdot T = 1$$

Angular speed, $\omega = \frac{Angle\ Covered}{Time\ Taken}$

$$\omega = \frac{2\pi}{T}$$

$$\boxed{\omega = 2\pi f}$$

Q7. Prove centripetal acceleration $= \frac{v^2}{r}$

Ans. "*Particle is moving in a circle with a uniform speed*".

Position vector triangle OAB is drawn as shown in fig. II Velocity vector triangle PQR is drawn as shown in fig. III

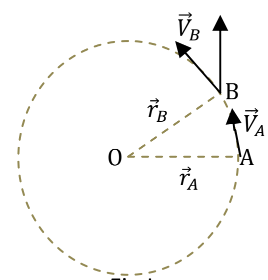


Fig.I

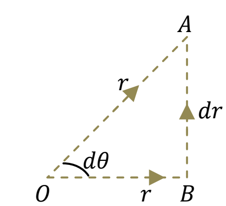
In fig. II $\quad d\theta = \frac{dr}{r}$ ------ (1)

In fig. III $\quad d\theta = \frac{dv}{v}$ ------ (2)

From (1) & (2)

$$\frac{dv}{v} = \frac{dr}{r}$$

$$\boxed{dv = \frac{v}{r}.\,dr} \quad \text{------ (3)}$$



Position Vector Triangle
Fig.II

As per definition

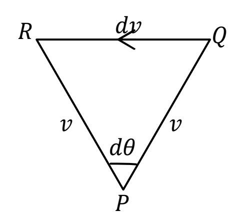
$$\text{acc, a} = \frac{dv}{dt}$$

$$= \frac{v}{r}\left(\frac{dr}{dt}\right)$$

$$= \frac{v}{r} \cdot v$$

$$a = \frac{v^2}{r}$$

$$\boxed{\text{centripetal acceleration } a_c = \frac{v^2}{r}}$$



Velocity Triangle
Fig.III

<u>Direction</u>:- $\quad \vec{a} = \frac{\overrightarrow{dv}}{dt}$

Direction of $\vec{a}$ is same as that of $\overrightarrow{dv}$ in fig.III

Direction of $\vec{a}$ is opposite to $\vec{r}$

i.e. towards Centre.

<u>Conclusion</u>:- $\boxed{a_c = \frac{v^2}{r} \text{ and towards centre as shown in fig IV.}}$

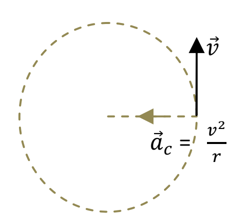
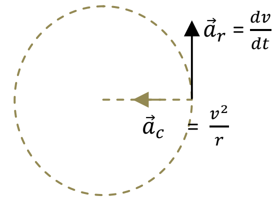


Fig.IV

Q8. **Write expression of $\vec{a}_c$ and $\vec{a}_r$ for "non uniform circular motion". (where a_c→centripetal, a_r→radial acc.)**



Ans. **Centripetal acceleration**,

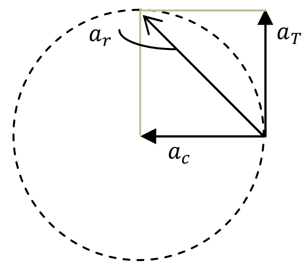
$$\vec{a}_c = \frac{v^2}{r}$$

Tangential acceleration,

$$\vec{a}_T = \frac{dv}{dt}$$

$$a_{res} = \sqrt{a_c^2 + a_T^2}$$

$$= \sqrt{\left(\frac{v^2}{r}\right)^2 + \left(\frac{dv}{dt}\right)^2}$$



Example:

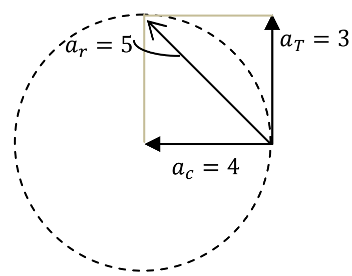
A particle is moving in a circle of radius 1 metre with it's speed increasing at 3m/sec in 1sec at an instant when its speed is 2m/sec. Find:

a) Centripetal acc.
b) Tangential acc.
c) Resultant acc.

Solution:

a) **Centripetal acc, a_c**

$$a_c = \frac{v^2}{r} = \frac{(2)^2}{1} = 4m/sec^2$$



b) **Tangential acc, a_T**

$$a_T = \frac{dv}{dt} = \frac{3m/sec}{1sec} = 3m/sec^2$$

c) **Resultant acc, $a_{resultant}$**

$$a_{resultant} = \sqrt{a_c^2 + a_T^2}$$

$$= \sqrt{4^2 + 3^2}$$

$$= 5m/sec^2$$